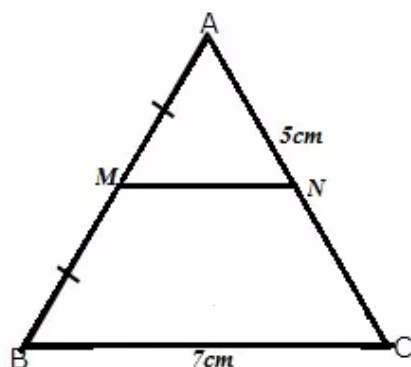


Chapter 12. Mid-point and Its Converse [Including Intercept Theorem]

Exercise 12(A)

Solution 1:

The triangle is shown below,



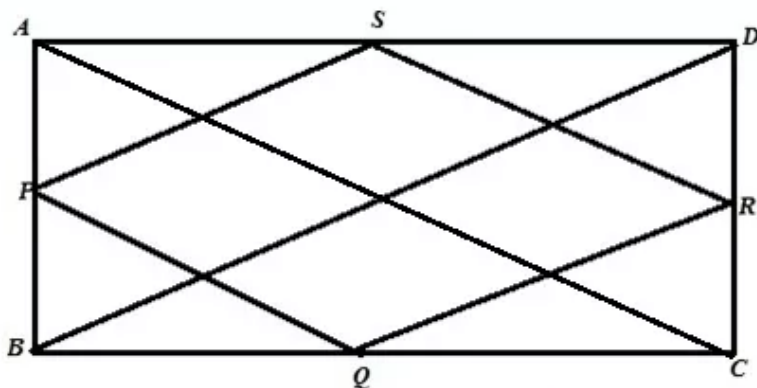
Since M is the midpoint of AB and $MN \parallel BC$ hence N is the midpoint of AC. Therefore

$$MN = \frac{1}{2} BC = \frac{1}{2} \times 7 = 3.5 \text{ cm}$$

$$\text{And } AN = \frac{1}{2} AC = \frac{1}{2} \times 5 = 2.5 \text{ cm}$$

Solution 2:

The figure is shown below,



Let ABCD be a rectangle where P, Q, R, S are the midpoint of AB, BC, CD, DA. We need to show that PQRS is a rhombus

For help we draw two diagonal BD and AC as shown in figure

Where $BD = AC$ (Since diagonal of rectangle are equal)

Proof:

From $\triangle ABD$ and $\triangle BCD$

$$PS = \frac{1}{2} BD = QR \text{ and } PS \parallel BD \parallel QR$$

$$2PS = 2QR = BD \text{ and } PS \parallel QR \quad \text{----- (1)}$$

$$\text{Similarly } 2PQ = 2SR = AC \text{ and } PQ \parallel SR \quad \text{----- (2)}$$

From (1) and (2) we get

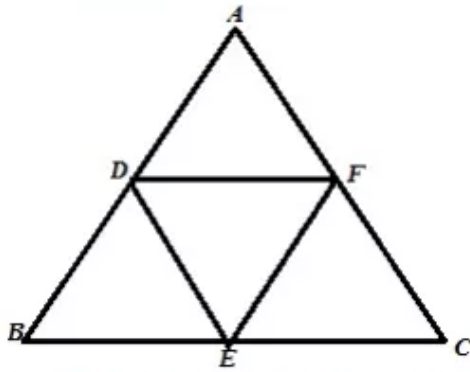
$$PQ = QR = RS = PS$$

Therefore PQRS is a rhombus.

Hence proved

Solution 3:

The figure is shown below



Given that ABC is an isosceles triangle where $AB=AC$.

Since D,E,F are midpoint of AB,BC,CA therefore

$2DE=AC$ and $2EF=AB$ this means $DE=EF$

Therefore DEF is an isosceles triangle and $DE=EF$.

Hence proved

Solution 4:

Here from triangle ABD P is the midpoint of AD and $PR \parallel AB$, therefore Q is the midpoint of BD

Similarly R is the midpoint of BC as $PR \parallel CD \parallel AB$

From triangle ABD $2PQ=AB$ (1)

From triangle BCD $2QR=CD$ (2)

Now (1)+(2)=>

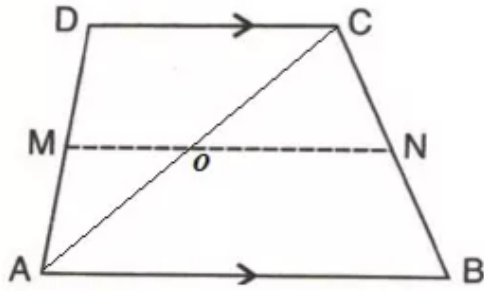
$2(PQ+QR)=AB+CD$

$$PR = \frac{1}{2}(AB + CD)$$

Hence proved

Solution 5:

Let we draw a diagonal AC as shown in the figure below,



(i) Given that $AB=11\text{cm}$, $CD=8\text{cm}$

From triangle ABC

$$ON = \frac{1}{2} AB = \frac{1}{2} \times 11 = 5.5\text{cm}$$

From triangle ACD

$$OM = \frac{1}{2} CD = \frac{1}{2} \times 8 = 4\text{cm}$$

Hence $MN=OM+ON=(4+5.5)=9.5\text{cm}$

(ii) Given that $CD=20\text{cm}$, $MN=27\text{cm}$

From triangle ACD

$$OM = \frac{1}{2} CD = \frac{1}{2} \times 20 = 10\text{cm}$$

Therefore $ON=27-10=17\text{cm}$

From triangle ABC

$$AB = 2ON = 2 \times 17 = 34\text{cm}$$

(iii) Given that $AB=23\text{cm}$, $MN=15\text{cm}$

From triangle ABC

$$ON = \frac{1}{2} AB = \frac{1}{2} \times 23 = 11.5\text{cm}$$

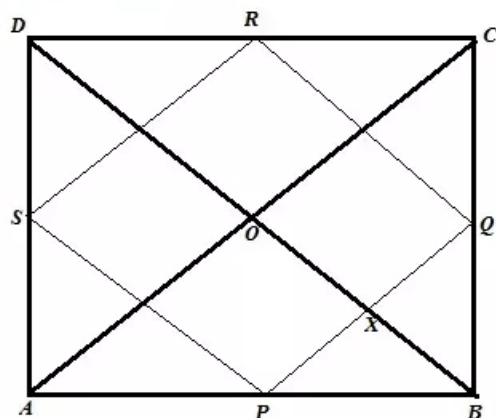
Therefore $OM=15-11.5=3.5\text{cm}$

From triangle ACD

$$CD = 2OM = 2 \times 3.5 = 7\text{cm}$$

Solution 6:

The figure is shown below



Let ABCD be a quadrilateral where P,Q,R,S are the midpoint of AB,BC,CD,DA. Diagonal AC and BD intersect at right angle at point O. We need to show that PQRS is a rectangle

Proof:

From $\triangle ABC$ and $\triangle ADC$

$2PQ=AC$ and $PQ \parallel AC$ (1)

$2RS=AC$ and $RS \parallel AC$ (2)

From (1) and (2) we get,

$PQ=RS$ and $PQ \parallel RS$

Similarly we can show that $PS=RQ$ and $PS \parallel RQ$

Therefore PQRS is a parallelogram.

Now $PQ \parallel AC$, therefore $\angle AOD = \angle PXO = 90^\circ$ [Corresponding angle]

Again $BD \parallel RQ$, therefore $\angle PXO = \angle RQX = 90^\circ$ [Corresponding angle]

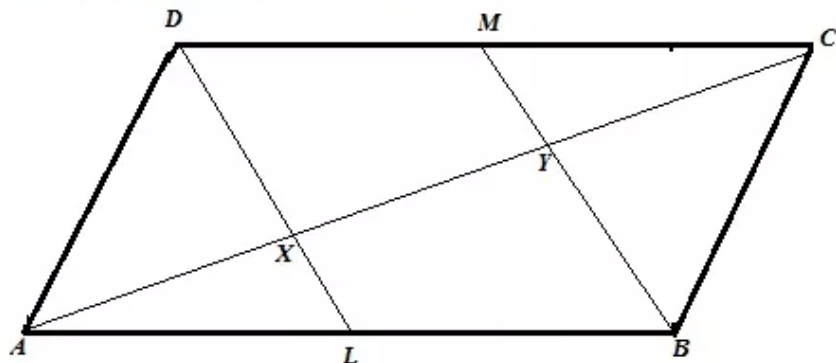
Similarly $\angle QRS = \angle RSP = \angle SPQ = 90^\circ$

Therefore PQRS is a rectangle.

Hence proved

Solution 7:

The required figure is shown below



From figure,

$BL=DM$ and $BL \parallel DM$ and BLMD is a parallelogram, therefore $BM \parallel DL$

From triangle ABY

L is the midpoint of AB and $XL \parallel BY$, therefore x is the midpoint of AY, ie $AX=XY$ (1)

Similarly for triangle CDX

$CY=XY$ (2)

From (1) and (2)

$AX=XY=CY$ and $AC=AX+XY+CY$

Hence proved

Solution 8:

Given that $AD=BC$ (1)

From the figure,

For triangle ADC and triangle ABD

$2GH=AD$ and $2EF=AD$, therefore $2GH=2EF=AD$ (2)

For triangle BCD and triangle ABC

$2GF=BC$ and $2EH=BC$, therefore $2GF=2EH=BC$ (3)

From (1),(2),(3) we get,

$2GH=2EF=2GF=2EH$

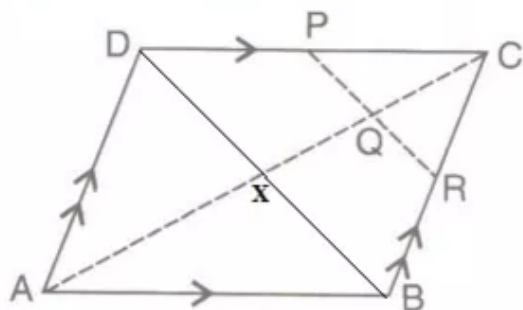
$GH=EF=GF=EH$

Therefore EFGH is a rhombus.

Hence proved

Solution 9:

For help we draw the diagonal BD as shown below



The diagonal AC and BD cuts at point X.

We know that the diagonal of a parallelogram intersects equally each other. Therefore

$AX=CX$ and $BX=DX$

Given,

$$CQ = \frac{1}{4} AC$$

$$CQ = \frac{1}{4} \times 2CX$$

$$CQ = \frac{1}{2} CX$$

Therefore Q is the midpoint of CX.

(i) For triangle CDX $PQ \parallel DX$ or $PR \parallel BD$

Since for triangle CBX

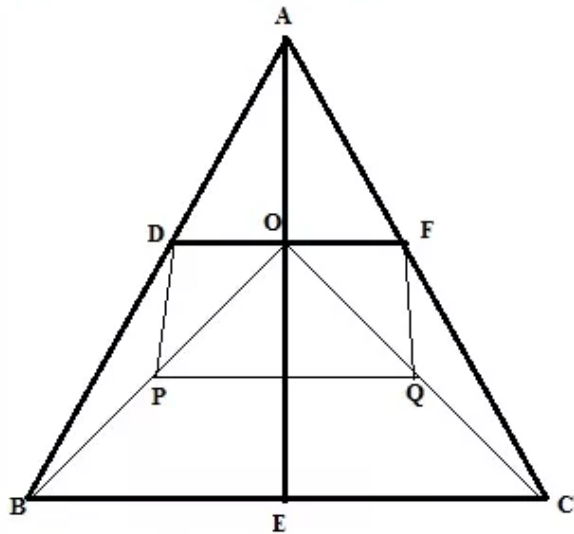
Q is the midpoint of CX and $QR \parallel BX$. Therefore R is the midpoint of BC

(ii) For triangle BCD

As P and R are the midpoint of CD and BC, therefore $PR = \frac{1}{2} DB$

Solution 10:

The required figure is shown below



For triangle ABC and OBC

$2DE=BC$ and $2PQ=BC$, therefore $DE=PQ$ (1)

For triangle ABO and ACO

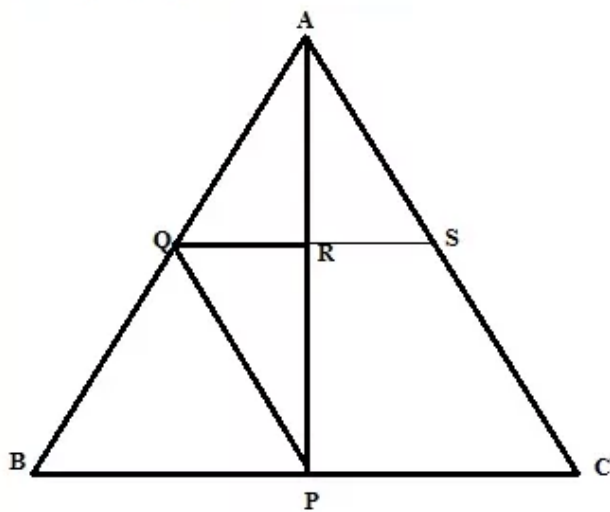
$2PD=AO$ and $2FQ=AO$, therefore $PD=FQ$ (2)

From (1),(2) we get that PQFD is a parallelogram.

Hence proved

Solution 11:

The required figure is shown below



From the figure it is seen that P is the midpoint of BC and $PQ \parallel AC$ and $QR \parallel BC$

Therefore Q is the midpoint of AB and R is the midpoint of AP

(i) Therefore $AP=2AR$

(ii) Here we increase QR so that it cuts AC at S as shown in the figure.

(iii) From triangle PQR and triangle ARS

$$\angle PQR = \angle ARS \quad (\text{Opposite angle})$$

$$PR = AR$$

$$PQ = AS \quad \left[PQ = AS = \frac{1}{2} AC \right]$$

$$\triangle PQR \cong \triangle ARS \quad (\text{SAS Postulate})$$

Therefore $QR=RS$

Now

$$BC = 2QS$$

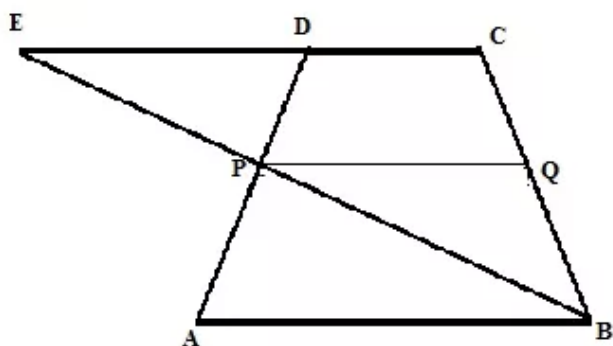
$$BC = 2 \times 2QR$$

$$BC = 4QR$$

Hence proved

Solution 12:

The required figure is shown below



(i)

From $\triangle PED$ and $\triangle ABP$

$$PD = AP \quad [P \text{ is the midpoint of } AD]$$

$$\angle DPE = \angle APB \quad [Opposite \text{ angle}]$$

$$\angle PED = \angle PBA \quad [AB \parallel CE]$$

$$\therefore \triangle PED \cong \triangle ABP \quad [ASA \text{ postulate}]$$

$$\therefore EP = BP$$

(ii) For triangle ECB $PQ \parallel CE$

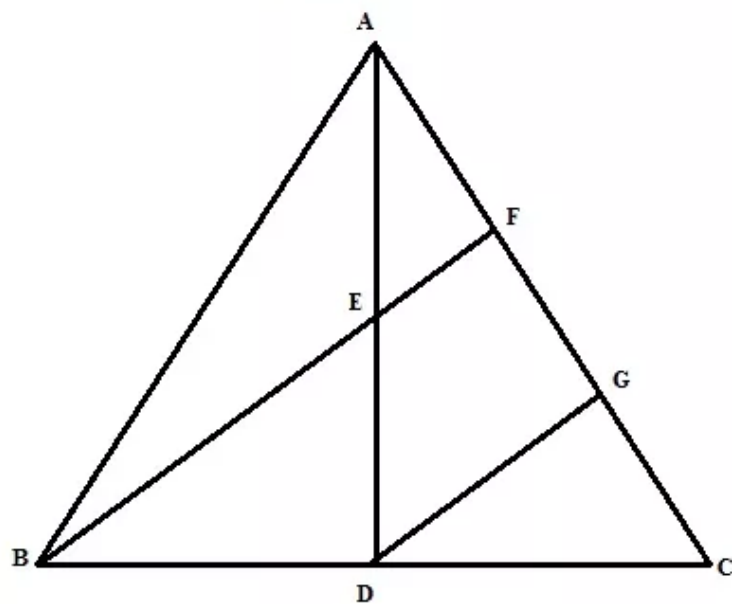
Again $CE \parallel AB$

Therefore $PQ \parallel AB$

Hence proved

Solution 13:

The required figure is shown below



For help we draw a line $DG \parallel BF$

Now from triangle ADG, $DG \parallel BF$ and E is the midpoint of AD

Therefore F is the midpoint of AG, i.e. $AF = GF$ (1)

From triangle BCF, $DG \parallel BF$ and D is the midpoint of BC

Therefore G is the midpoint of CF, i.e. $GF = CF$... (2)

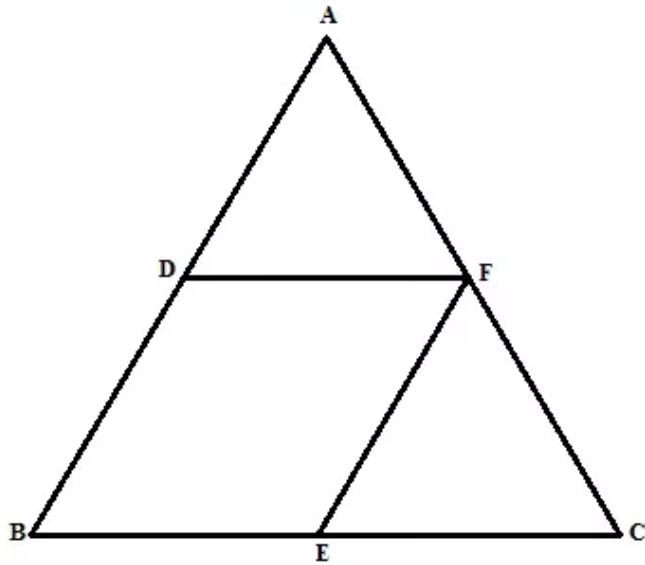
$$AC = AF + GF + CF$$

$$AC = 3AF \text{ (From (1) and (2))}$$

Hence proved

Solution 14:

The required figure is shown below



(i) Since F is the midpoint and $EF \parallel AB$.

Therefore E is the midpoint of BC

$$\text{So } BE = \frac{1}{2}BC \text{ and } EF = \frac{1}{2}AB \dots (1)$$

Since D and F are the midpoint of AB and AC

Therefore $DE \parallel BC$

$$\text{So } DF = \frac{1}{2}BC \text{ and } DB = \frac{1}{2}AB \dots (2)$$

From (1), (2) we get

$$BE = DF \text{ and } BD = EF$$

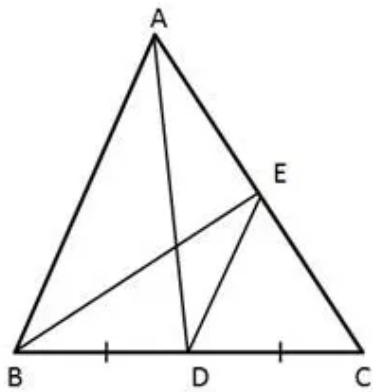
Hence BDEF is a parallelogram.

(ii) Since

$$AB = 2EF$$

$$= 2 \times 4.8$$

$$= 9.6 \text{ cm}$$

Solution 15:

In $\triangle ABC$,

AD is the median of BC

\Rightarrow D is the mid-point of BC.

Given that $DE \parallel AB$

By the Converse of the Mid-point theorem,

\Rightarrow DE bisects AC

\Rightarrow E is the mid-point of AC

\Rightarrow BE is the median of AC,

that is BE is also a median.

Solution 16:

Construction : Draw $DY \parallel BQ$

In $\triangle BCQ$ and $\triangle DCY$,

$$\angle BCQ = \angle DCY \text{ (Common)}$$

$$\angle BQC = \angle DYC \text{ (Corresponding angles)}$$

So, $\triangle BCQ \sim \triangle DCY$ (AA Similarity criterion)

$$\Rightarrow \frac{BQ}{DY} = \frac{BC}{DC} = \frac{CQ}{CY} \text{ (Corresponding sides are proportional)}$$

$$\Rightarrow \frac{BQ}{DY} = \frac{2CD}{CD} \text{ (D is the mid - point of BC)}$$

$$\Rightarrow \frac{BQ}{DY} = 2 \dots (i)$$

Similarly, $\triangle AEQ \sim \triangle ADY$

$$\Rightarrow \frac{EQ}{DY} = \frac{AE}{ED} = \frac{1}{2} \text{ (E is the mid - point of AD)}$$

$$\text{that is } \frac{EQ}{DY} = \frac{1}{2} \dots (ii)$$

Dividing (i) by (ii), we get

$$\Rightarrow \frac{BQ}{EQ} = 4$$

$$\Rightarrow BE + EQ = 4EQ$$

$$\Rightarrow BE = 3EQ$$

$$\Rightarrow \frac{BE}{EQ} = \frac{3}{1}$$

Solution 17:

In $\triangle EDF$,

M is the mid - point of AB and N is the mid - point of DE.

$$\Rightarrow MN = \frac{1}{2} EF \text{ (Mid - point theorem)}$$

$$\Rightarrow EF = 2MN \dots (i)$$

In $\triangle ABC$,

M is the mid - point of AB and N is the mid - point of BC.

$$\Rightarrow MN = \frac{1}{2} AC \text{ (Mid - point theorem)}$$

$$\Rightarrow AC = 2MN \dots (ii)$$

From (i) and (ii), we get

$$\Rightarrow EF = AC$$

Exercise 12(B)**Solution 1:**

According to equal intercept theorem since $CD = DE$

Therefore $AB = BC$ and $EF = GF$

$$(i) BC = AB = 7.2 \text{ cm}$$

$$(ii) GE = EF + GF = 2EF = 2 \times 4 = 8 \text{ cm}$$

Since B, D, F are the midpoint and $AE \parallel BF \parallel CG$

Therefore $AE = 2BD$ and $CG = 2DF$

$$(iii) AE = 2BD = 2 \times 4.1 = 8.2$$

$$(iv) DF = \frac{1}{2} CG = \frac{1}{2} \times 11 = 5.5 \text{ cm}$$

Solution 2:

Given that $AD=AP=PB$ as $2AD=AB$ and p is the midpoint of AB

(i) From triangle DPR , A and Q are the midpoint of DP and DR .

Therefore $AQ \parallel PR$

Since $PR \parallel BS$, hence $AQ \parallel BS$

(ii) From triangle ABC , P is the midpoint and $PR \parallel BS$

Therefore R is the midpoint of BC

From $\triangle BRS$ and $\triangle QRC$

$$\angle BRS = \angle QRC$$

$$BR = RC$$

$$\angle RBS = \angle RCQ$$

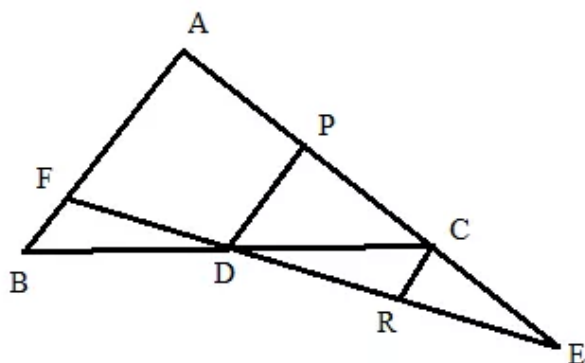
$$\therefore \triangle BRS \cong \triangle QRC$$

$$\therefore QR = RS$$

$$DS = DQ + QR + RS = QR + QR + RS = 3RS$$

Solution 3:

Consider the figure:



Here D is the midpoint of BC and DP is parallel to AB , therefore P is the midpoint of AC and $PD = \frac{1}{2}AB$

(i)

Again from the triangle AEF we have $AE \parallel PD \parallel CR$ and $AP = \frac{1}{3}AE$

Therefore $DF = \frac{1}{3}EF$ or we can say that $3DF = EF$.

Hence it is shown.

(ii)

From the triangle PED we have $PD \parallel CR$ and C is the midpoint of PE therefore $CR = \frac{1}{2}PD$

Now

$$PD = \frac{1}{2}AB$$

$$\frac{1}{2}PD = \frac{1}{4}AB$$

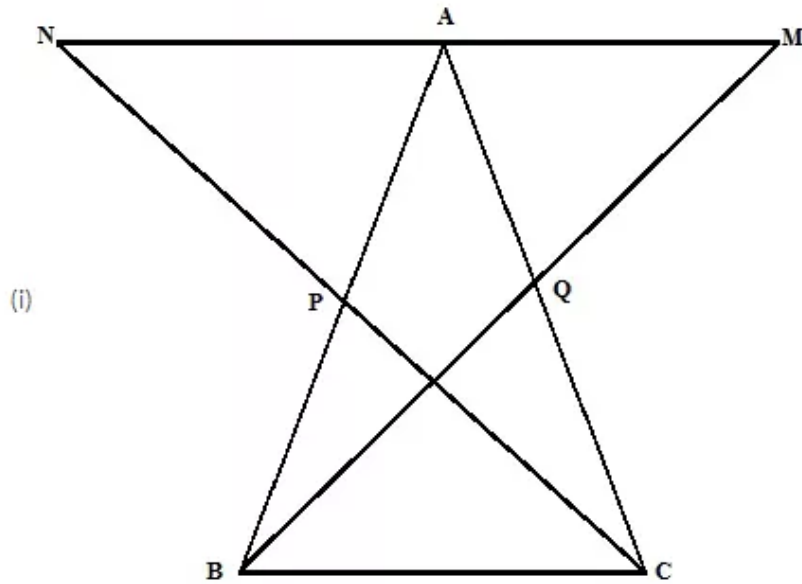
$$CR = \frac{1}{4}AB$$

$$4CR = AB$$

Hence it is shown.

Solution 4:

The figure is shown below



From triangle BPC and triangle APN

$$\angle BPC = \angle APN \quad [\text{Opposite angle}]$$

$$BP = AP$$

$$PC = PN$$

$$\therefore \triangle BPC \cong \triangle APN \quad [\text{SAS postulate}]$$

$$\therefore \angle PBC = \angle PAN \quad \dots\dots (1)$$

$$\text{And } BC = AN \quad \dots\dots (3)$$

$$\text{Similarly } \angle QCB = \angle QAN \quad \dots\dots (2)$$

$$\text{And } BC = AM \quad \dots\dots (4)$$

Now

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

$$\angle PAN + \angle QAM + \angle BAC = 180^\circ \quad [(1), (2) \text{ we get}]$$

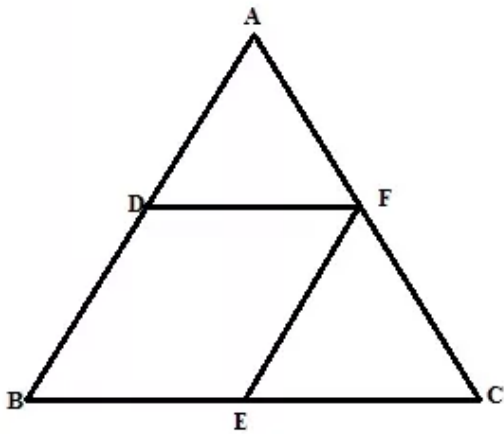
Therefore M, A, N are collinear

(ii) From (3) and (4) $MA = NA$

Hence A is the midpoint of MN

Solution 5:

The figure is shown below



From the figure $EF \parallel AB$ and E is the midpoint of BC.

Therefore F is the midpoint of AC.

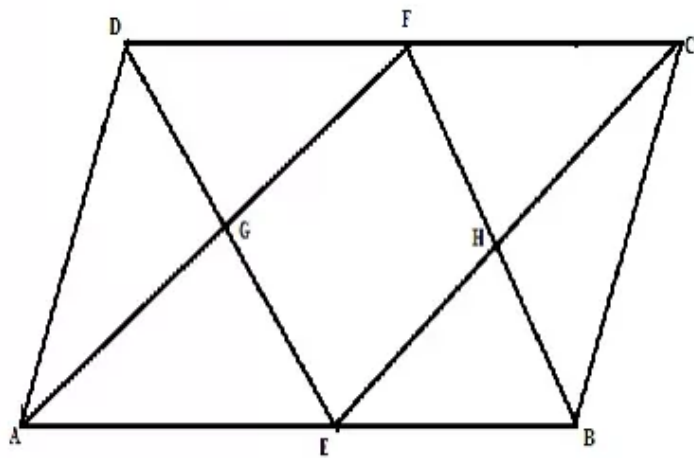
Here $EF \parallel BD$, $EF = BD$ as D is the midpoint of AB

$BE \parallel DF$, $BE = DF$ as E is the midpoint of BC.

Therefore BEFD is a parallelogram.

Solution 6:

The figure is shown below



(i)

From $\triangle HEB$ and $\triangle FHC$

$$BE = FC$$

$$\angle EHB = \angle FHC \quad [\text{Opposite angle}]$$

$$\angle HBE = \angle HFC$$

$$\therefore \triangle HEB \cong \triangle FHC$$

$$\therefore EH = CH, BH = FH$$

(ii)

Similarly $AG = GF$ and $EG = DG$ (1)

For triangle ECD, F and H are the midpoint of CD and EC.

Therefore $HF \parallel DE$ and $HF = \frac{1}{2} DE$ (2)

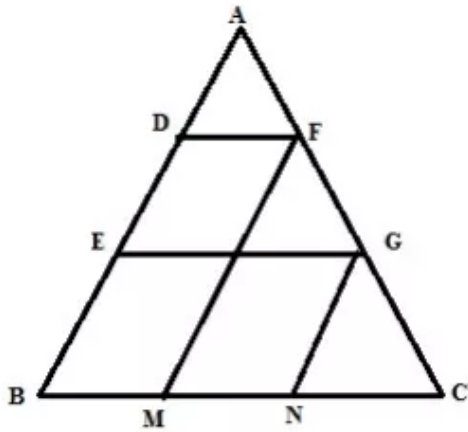
(1),(2) we get, $HF = EG$ and $HF \parallel EG$

Similarly we can show that $EH = GF$ and $EH \parallel GF$

Therefore GEHF is a parallelogram.

Solution 7:

The figure is shown below



For triangle AEG

D is the midpoint of AE and $DF \parallel EG \parallel BC$

Therefore F is the midpoint of AG.

$$AF = GF \dots (1)$$

Again $DF \parallel EG \parallel BC$ $DE = BE$, therefore $GF = GC \dots (2)$

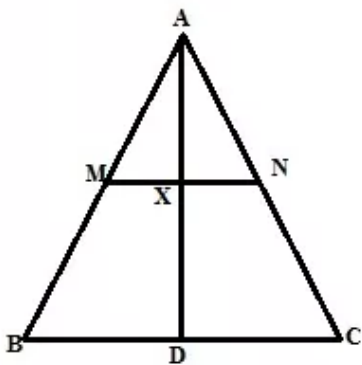
(1), (2) we get $AF = GF = GC$.

Similarly Since $GN \parallel FM \parallel AB$ and $AF = GF$, therefore $BM = MN = NC$

Hence proved

Solution 8:

The figure is shown below



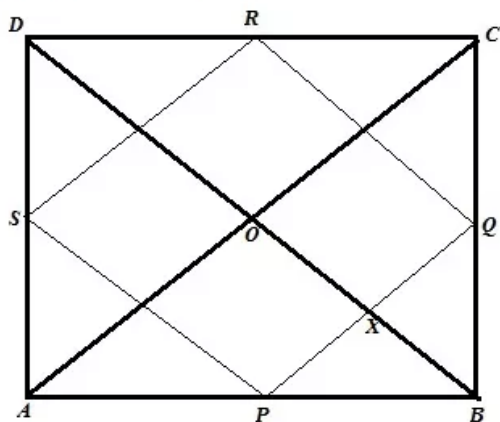
Since M and N are the midpoint of AB and AC, $MN \parallel BC$

According to intercept theorem Since $MN \parallel BC$ and $AM = BM$,

Therefore $AX = DX$. Hence proved

Solution 9:

The figure is shown below



Let ABCD be a quadrilateral where P,Q,R,S are the midpoint of AB,BC,CD,DA. PQRS is a rectangle. Diagonal AC and BD intersect at point O. We need to show that AC and BD intersect at right angle.

Proof:

$PQ \parallel AC$, therefore $\angle AOD = \angle PXO$ [Corresponding angle](1)

||Again $BD \parallel RQ$, therefore $\angle PXO = \angle RQX = 90^\circ$ [Corresponding angle and angle of rectangle] ...(2)

From (1) and (2) we get ,

$\angle AOD = 90^\circ$

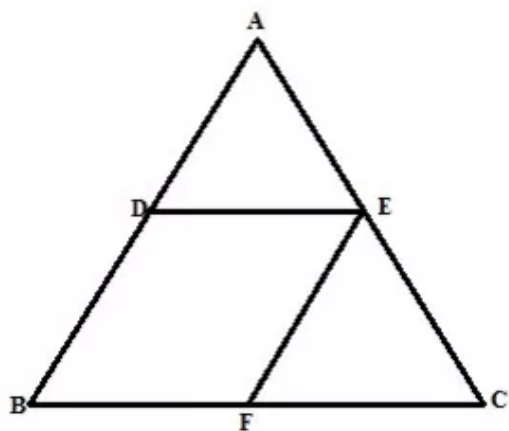
Similarly $\angle AOB = \angle BOC = \angle DOC = 90^\circ$

Therefore diagonals AC and BD intersect at right angle

Hence proved

Solution 10:

The figure is shown below



From figure since E is the midpoint of AC and $EF \parallel AB$

Therefore F is the midpoint of BC and $2DE = BC$ or $DE = BF$

Again D and E are midpoint ,therefore $DE \parallel BF$ and $EF = BD$

Hence BDEF is a parallelogram.

Now

$$BD = EF = \frac{1}{2} AB = \frac{1}{2} \times 16 = 8cm$$

$$BF = DE = \frac{1}{2} BC = \frac{1}{2} \times 18 = 9cm$$

Therefore perimeter of BDEF = $2(BF + EF) = 2(9 + 8) = 34cm$

Solution 11:

Given AD and CE are medians and $DF \parallel CE$.

We know that from the midpoint theorem, if two lines are parallel and the starting point of segment is at the midpoint on one side, then the other point meets at the midpoint of the other side.

Consider triangle BEC. Given $DF \parallel CE$ and D is midpoint of BC.

So F must be the midpoint of BE.

$$\text{So } FB = \frac{1}{2}BE \text{ but } BE = \frac{1}{2}AB$$

Substitute value of BE in first equation, we get

$$FB = \frac{1}{4}AB$$

Hence Prove

Solution 12:

Given ABCD is parallelogram, so $AD = BC$, $AB = CD$.

Consider triangle APB, given EC is parallel to AP and E is midpoint of side AB. So by midpoint theorem, C has to be the midpoint of BP.

So $BP = 2BC$, but $BC = AD$ as ABCD is a parallelogram.

Hence $BP = 2AD$

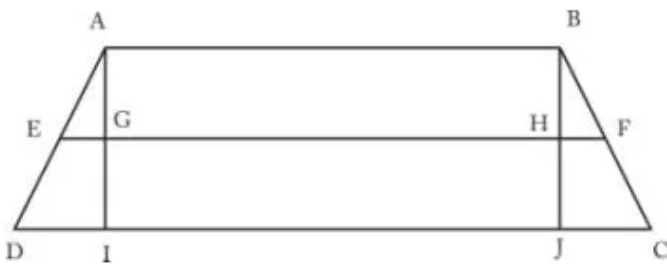
Consider triangle APB, $AB \parallel OC$ as ABCD is a parallelogram. So by midpoint theorem, O has to be the midpoint of AP.

Hence Proved

Solution 13:

Consider trapezium ABCD.

Given E and F are midpoints on sides AD and BC, respectively.



We know that $AB = GH = IJ$

From midpoint theorem, $EG = \frac{1}{2}DI$, $HF = \frac{1}{2}JC$

Consider LHS,

$$AB + CD = AB + CJ + JI + ID = AB + 2HF + AB + 2EG$$

$$\text{So } AB + CD = 2(AB + HF + EG) = 2(EG + GH + HF) = 2EF$$

$$AB + CD = 2EF$$

Hence Proved

Solution 14:

Given ΔABC

AD is the median. So D is the midpoint of side BC.

Given $DE \parallel AB$. By the midpoint theorem, E has to be midpoint of AC.

So line joining the vertex and midpoint of the opposite side is always known as median. So BE is also median of ΔABC .